

# Detecting perfusion abnormalities in Arterial Spin Labelling

## Random-effects and Mixed-effects GLM

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NeuroImaging seminars, Institute of Psychiatry  
King's College London - February 14<sup>th</sup>, 2014

# Outline

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Detecting  
perfusion  
abnormalities

ASL

Group

Single-subject

General Linear  
Model: MFX and  
RFX

Hypothesis  
testing

RFX and MFX

Detections in ASL

Methods

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Results

Conclusions

Detecting perfusion abnormalities

Random-Effects and Mixed-Effects General Linear Model

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Single-subject analysis

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## Random-Effects and Mixed-Effects General Linear Model

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#### Random-effects and Mixed-effects

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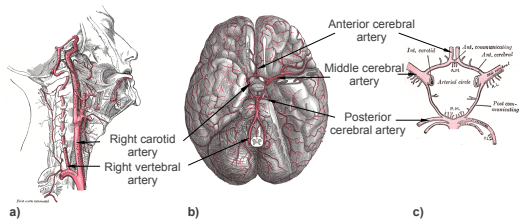
#### Experiment

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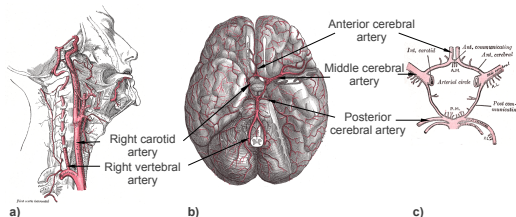


# Brain perfusion



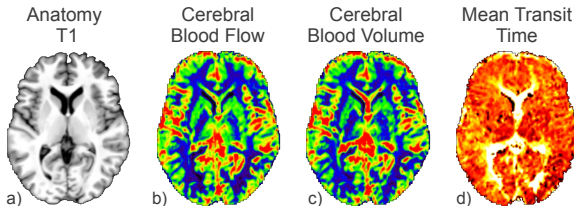
Blood supply to the brain

# Brain perfusion



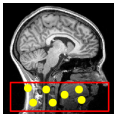
Blood supply to the brain

**Brain perfusion** is the biological process that ensures the delivery of oxygen and nutrients to the cerebral tissues by means of microcirculation.



Example of perfusion parameters

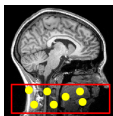
# Arterial Spin Labelling (ASL)



Labelling

[Detre et al., MRM 1992]

# Arterial Spin Labelling (ASL)



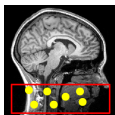
Labelling



Delay

[Detre et al., MRM 1992]

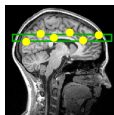
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Labelling



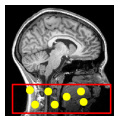
Delay



Readout

[Detre et al., MRM 1992]

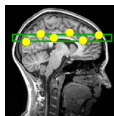
# Arterial Spin Labelling (ASL)



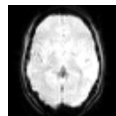
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Delay



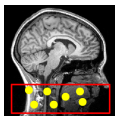
Readout



Label

[Detre et al., MRM 1992]

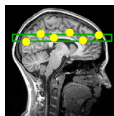
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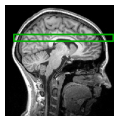
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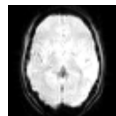
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Readout



Readout



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[Detre et al., MRM 1992]

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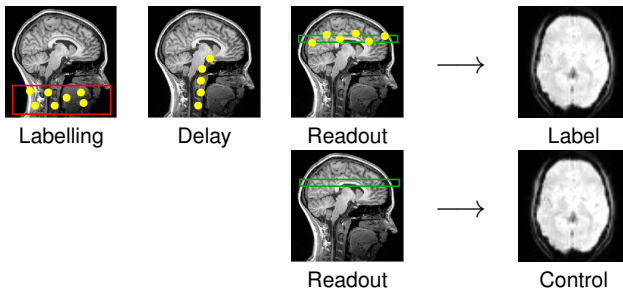
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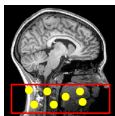
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[Detre et al., MRM 1992]



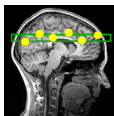
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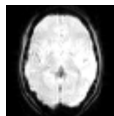
Labelling



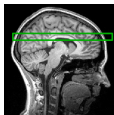
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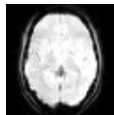
Readout



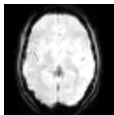
Label



Readout

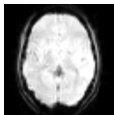


Control



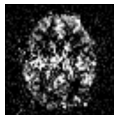
Control

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Label

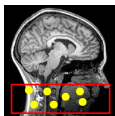
=



Difference  
1 pair

[Detre et al., MRM 1992]

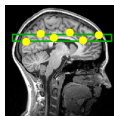
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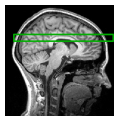
Labelling



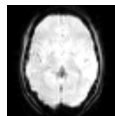
Delay



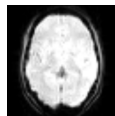
Readout



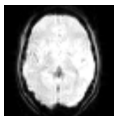
Readout



Label

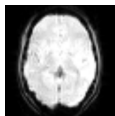


Control



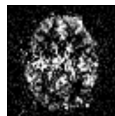
Control

—



Label

=



Difference  
1 pair



Difference  
60 pairs

[Detre et al., MRM 1992]

# Outline

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Arterial Spin Labelling

Group analysis

Single-subject analysis

## Random-Effects and Mixed-Effects General Linear Model

Hypothesis testing

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## Detections in ASL

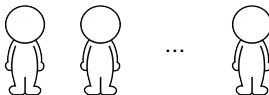
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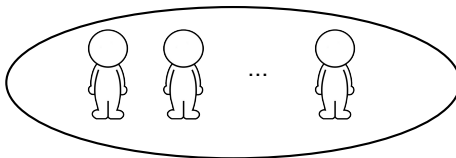
## Conclusions

# Within-group analysis



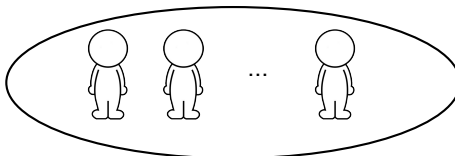
# Within-group analysis

Identify common patterns across a group of subjects.

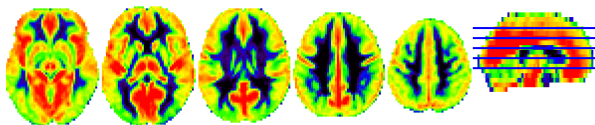


# Within-group analysis

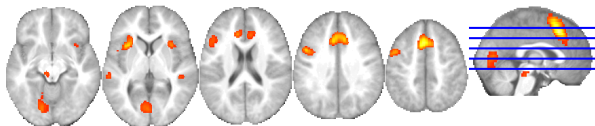
Identify common patterns across a group of subjects.



## Examples

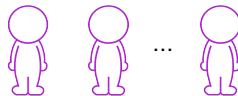


Group cerebral blood flow.



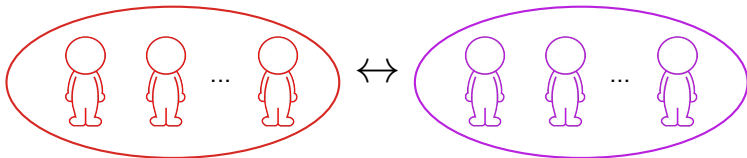
Group activation for a language task.

# Between-group analysis



# Between-group analysis

Identify differences at the group level.



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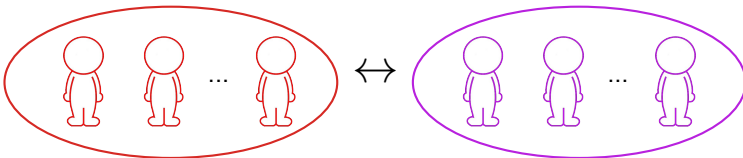
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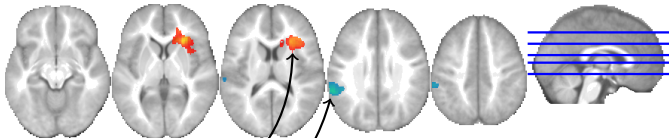


# Between-group analysis

Identify differences at the group level.



## Example



Differences of perfusion between a group of patients and a control group.

- ▶ Hyper-perfusion.
- ▶ Hypo-perfusion.

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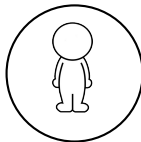
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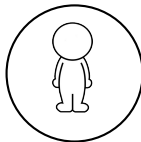
# Within-subject analysis

Identify patterns of perfusion (or activation) in a single subject.

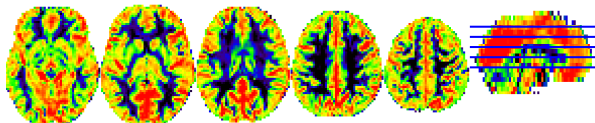


# Within-subject analysis

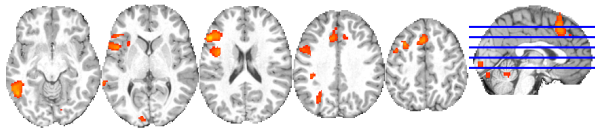
Identify patterns of perfusion (or activation) in a single subject.



## Examples



Cerebral blood flow.



Subject activation for a language task.

# Between-group individual analysis

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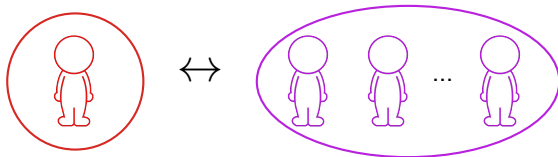
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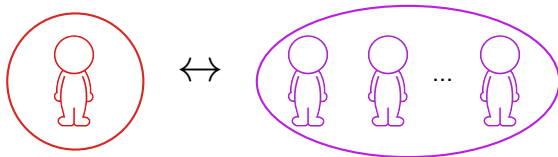
# Between-group individual analysis

Identify deviation from normality in a single subject.

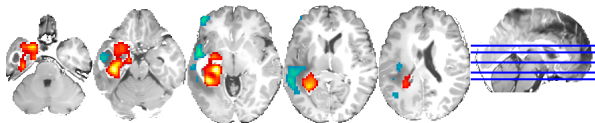


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Identify deviation from normality in a single subject.



## Example

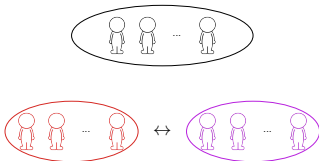


Hyper- and hypo-perfusions in a patient diagnosed with brain tumour.



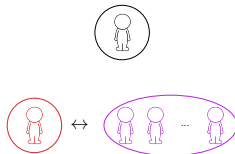
# Within-group and between-group analyses

## Group analyses



- ▶ Study of **typical brain perfusion**.
- ▶ Provide a better understanding of **brain dysfunction** associated with a pathology.

## Individual analyses



- ▶ Study of **brain perfusion in a particular subject**.
- ▶ Outline **deviation from normality** (or from a reference group).

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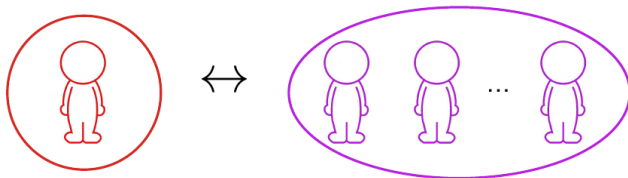
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# A massively univariate approach



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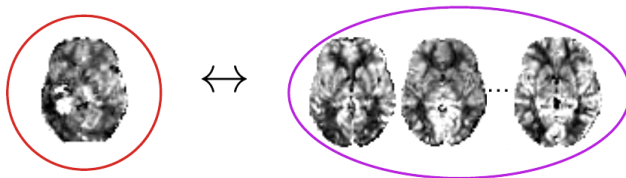
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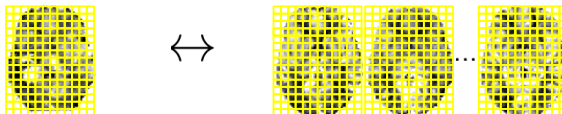
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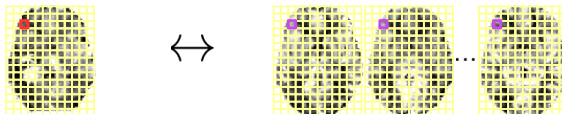
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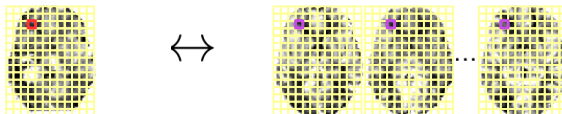
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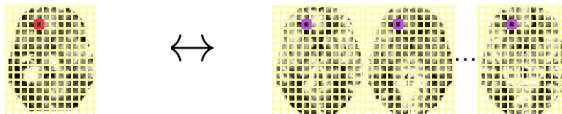
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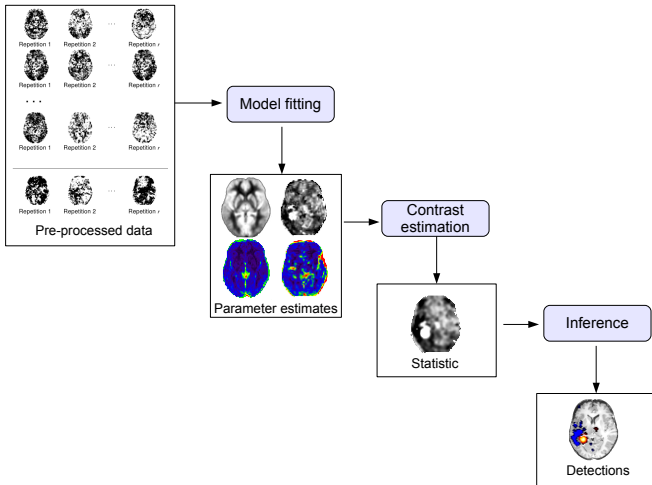
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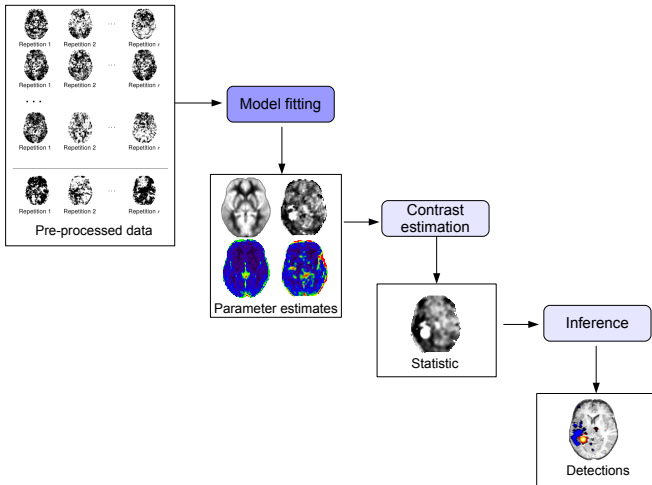
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## Model fitting

Using the GLM, the dataset of interest is modelled as a linear combination of pre-defined parameters.

$$\begin{matrix} \xrightarrow{1} \\ \downarrow N \end{matrix} y = \begin{matrix} \xrightarrow{p} \\ \downarrow N \end{matrix} X \begin{matrix} \xrightarrow{1} \\ \downarrow p \end{matrix} \beta + \begin{matrix} \xrightarrow{1} \\ \downarrow N \end{matrix} e$$

Source: "The General Linear Model for fMRI analyses" by FIL Methods Group, SPM Course, 2013.

## Model fitting

Using the GLM, the dataset of interest is modelled as a linear combination of pre-defined parameters.

$$y = X\beta + e$$

The diagram illustrates the dimensions of the variables in the GLM equation. The vector  $y$  is  $N \times 1$ . The matrix  $X$  is  $N \times p$ . The vector  $\beta$  is  $p \times 1$ . The vector  $e$  is  $N \times 1$ .

Source: "The General Linear Model for fMRI analyses" by FIL Methods Group, SPM Course, 2013.

Independent and identically distributed errors, i.e. given  $e \sim \mathcal{N}(0, \sigma^2 I)$ , we can use Ordinary Least Squares:

$$\hat{\beta}_{OLS} = (X^T X)^{-1} X^T Y, \quad \widehat{\text{Var}}(\hat{\beta}_{OLS}) = \hat{\sigma}^2 (X^T X)^{-1}. \quad (1)$$

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Otherwise, given  $e \sim \mathcal{N}(0, V)$ , Weighted Least Squares:

$$\hat{\beta}_{WLS} = (X^T V^{-1} X)^{-1} X^T V^{-1} Y, \quad \widehat{\text{Var}}(\hat{\beta}_{WLS}) = (X^T V^{-1} X)^{-1}. \quad (2)$$

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Single-subject

General Linear  
Model: MFX and  
RFX

Hypothesis  
testing

RFX and MFX

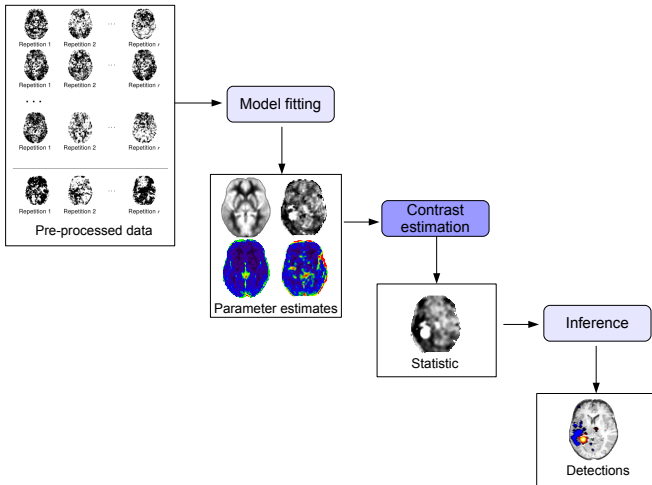
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# Hypothesis testing

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Under the null hypothesis:

$$H_0 : \mathbf{c}\beta_{(v)} = 0.$$

Assuming the normality of the error, the t-statistic at voxel  $v$  is defined by:

$$\frac{\mathbf{c}\hat{\beta}_{(v)}}{\sqrt{\widehat{\text{Var}}(\mathbf{c}\hat{\beta}_{(v)})}} \sim \mathcal{T}_{N-p}. \quad (3)$$



# Hypothesis testing: overview

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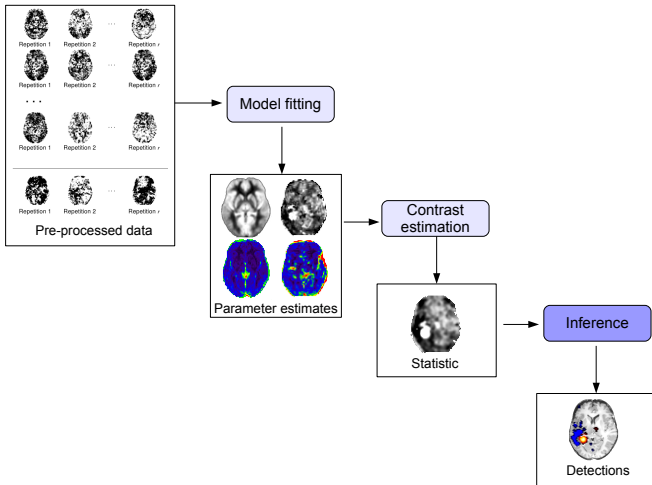
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# Outline

## Detecting perfusion abnormalities

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## Random-Effects and Mixed-Effects General Linear Model

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# Modelling: subject level



Repetition 1



Repetition 2

...



Repetition  $r$

# Modelling: subject level



# Modelling: subject level

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RFX and MFX

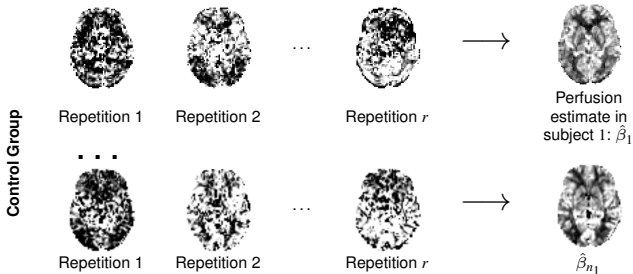
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# Modelling: subject level

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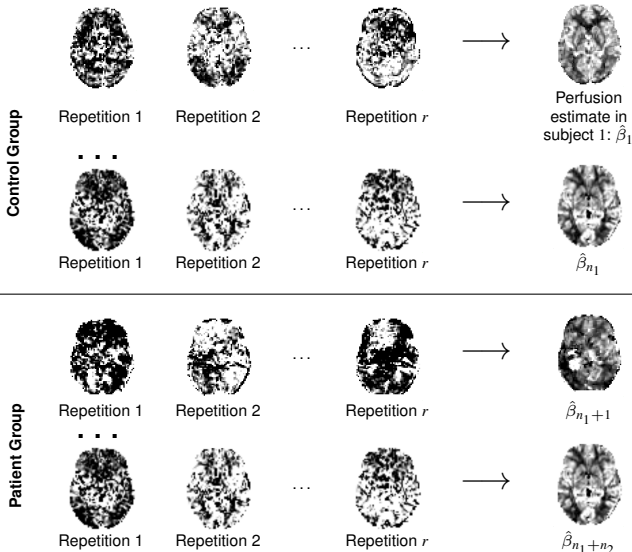
Detections in ASL

Methods

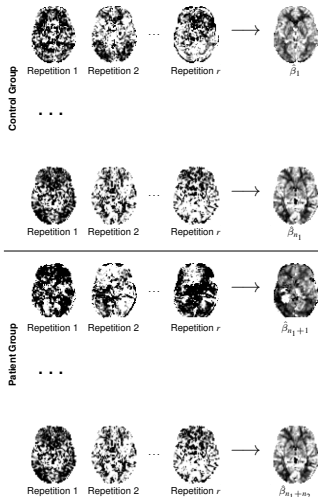
Experiment

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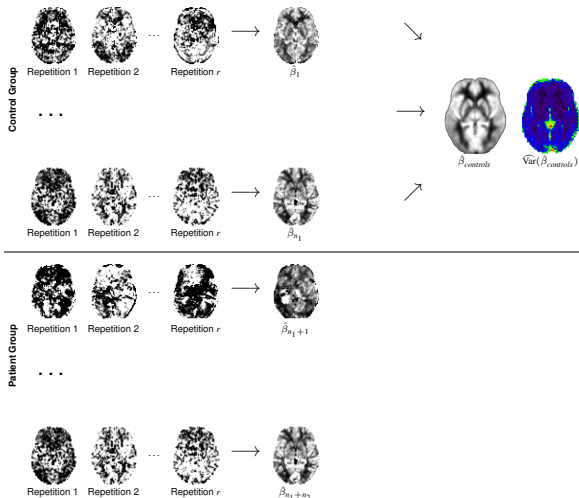
Conclusions



# Modelling: group level

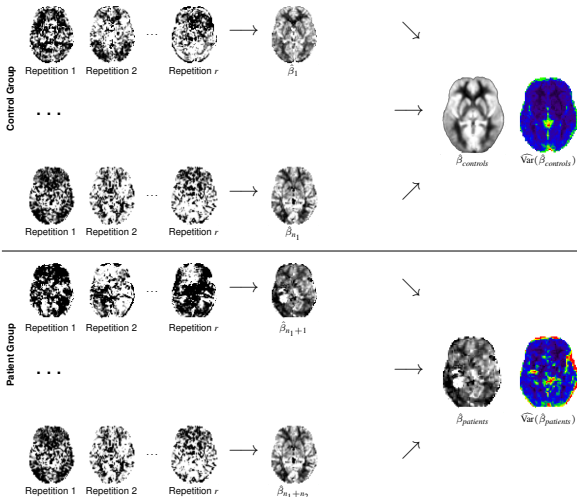


# Modelling: group level

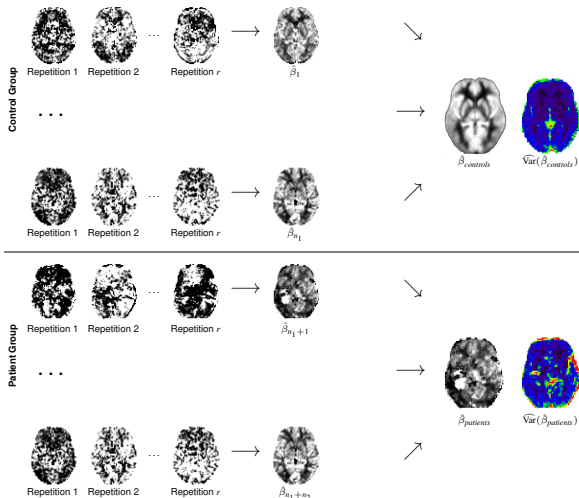




# Modelling: group level

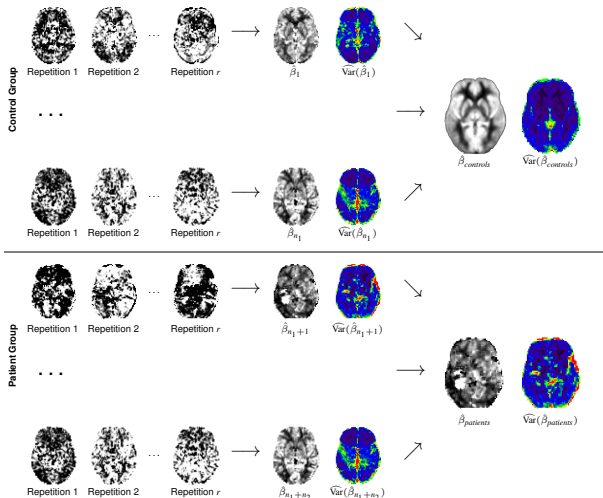


# Modelling: group level



**Random-effects (RFX) analysis.**

# Modelling: group level



**Mixed-effects (MFX) analysis.**

# Random-effects assumptions

**RFX analyses assume** that the **within-subject variance** is:

- ▶ **negligible** by comparison to the between-subject variance; or
- ▶ **roughly constant** across subjects.

# Random-effects or Mixed-effects analyses

In functional MRI there is no consensus:

- ▶ Superiority of MFX, [Beckmann 2003, Mumford 2006, Thirion 2007].
- ▶ Validity of RFX for one-sample t-tests in BOLD fMRI, [Mumford 2009].
- ▶ Invalidity of RFX [Chen 2012].

Both approaches are in use in the neuroimaging community:

- ▶ Random-effects analyses (SPM<sup>1</sup>)
- ▶ Mixed-effects analyses (FSL<sup>2</sup>, AFNI<sup>3</sup>).

## What about ASL?

---

<sup>1</sup>[www.fil.ion.ucl.ac.uk/spm/](http://www.fil.ion.ucl.ac.uk/spm/)

<sup>2</sup>[fsl.fmrib.ox.ac.uk/fsl/fslwiki/](http://fsl.fmrib.ox.ac.uk/fsl/fslwiki/)

<sup>3</sup>[afni.nimh.nih.gov/afni/](http://afni.nimh.nih.gov/afni/)

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## Detecting perfusion abnormalities

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## Random-Effects and Mixed-Effects General Linear Model

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# Detecting perfusion abnormalities using the GLM

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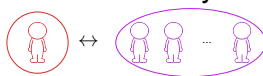
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Between-group analyses



Individual analyses



- Modelling and estimation using the GLM.
- Difference between random-effects and mixed-effects analyses.

Note: For ease of calculation, the models will be presented in the following without covariates.



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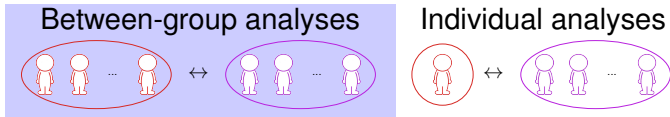
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- Modelling and estimation using the GLM.
- Difference between random-effects and mixed-effects analyses.

Note: For ease of calculation, the models will be presented in the following without covariates.

# GLM: subject level



Repetition 1



Repetition 2

...



Repetition  $r$

# GLM: subject level

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perfusion  
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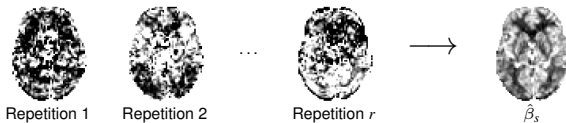
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# GLM: subject level



# GLM: subject level

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Given a voxel, for each subject  $s$  we have:

$$Y_s = X_s \beta_s + \epsilon_s, \quad (4)$$

where

- ▶  $Y_s$  vector of observations;
- ▶  $X_s$  subject-level design matrix;
- ▶  $\beta_s$  parameters to be estimated;
- ▶  $\epsilon_s$  residual error

## GLM: subject level



Given a voxel, for each subject  $s$  we have:

$$Y_s = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \beta_s + \epsilon_s. \quad (5)$$

where

- ▶  $Y_s$  vector of observations;
- ▶  $X_s$  subject-level design matrix;
- ▶  $\beta_s$  parameters to be estimated;
- ▶  $\epsilon_s$  residual error,  $\epsilon_s \sim \mathcal{N}(0, \sigma_s^2)$ . [Aguirre 2002]

# GLM: subject level



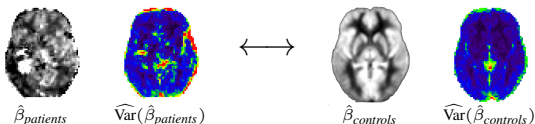
Assuming  $\epsilon_s \sim \mathcal{N}(0, \sigma_s^2)$ , by ordinary least squares we have:

$$\hat{\beta}_s = \frac{1}{r} \sum_{i=1}^r y_{s,i}, \text{ and } \widehat{\text{Var}}(\hat{\beta}_s) = \frac{\hat{\sigma}_s^2}{r}$$

where

- ▶  $y_{s,i}$  is the  $i^{\text{th}}$  element of vector  $Y_s$
- ▶  $\hat{\sigma}_s^2$  the estimated within-subject variance.

## GLM: between-group



The subject parameters  $(\beta_s)_{1 \leq s \leq n_1+1}$  can be combined using:

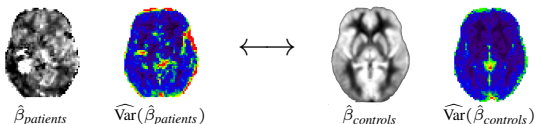
$$\begin{bmatrix} \beta_1 \\ \vdots \\ \beta_{n_1+n_2} \end{bmatrix} = X_G \beta_G + \gamma_G, \quad (6)$$

where

- ▶  $X_G$  is the group-level design matrix;
- ▶  $\beta_G$  the group parameters;
- ▶  $\gamma_G^s$  the residual error term.



## GLM: between-group



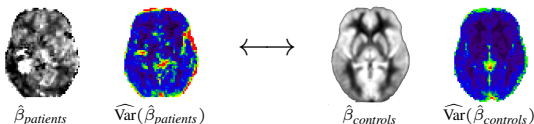
The subject parameters  $(\beta_s)_{1 \leq s \leq n_1+1}$  can be combined using:

$$\begin{bmatrix} \beta_1 \\ \vdots \\ \beta_{n_1} \\ \beta_{n_1+1} \\ \vdots \\ \beta_{n_1+n_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_{controls} \\ \beta_{patients} \end{bmatrix} + \gamma_G. \quad (7)$$

where

- $\gamma_G$  the residual error term,  $\gamma_G^s \sim \mathcal{N}(0, \sigma_{G,i}^2)$ .

## GLM: between-group



The subject parameters  $(\beta_s)_{1 \leq s \leq n_1+1}$  can be combined using:

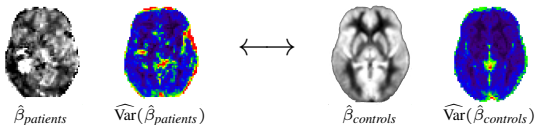
$$\begin{bmatrix} \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_{n_1} \\ \hat{\beta}_{n_1+1} \\ \vdots \\ \hat{\beta}_{n_1+n_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_{controls} \\ \beta_{patients} \end{bmatrix} + \gamma_{G_C}. \quad (8)$$

where

- $\gamma_{G_C}$  the residual error term,  $\gamma_{G_C}^s \sim \mathcal{N}\left(0, \sigma_{G,i}^2 + \frac{\sigma_s^2}{r}\right)$ .

# Contrast of interest

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We are interested in the null hypothesis:

$$H_0 : \beta_{controls} = \beta_{patients}. \quad (9)$$

$$H_0 : c \beta_G = 0. \quad (10)$$

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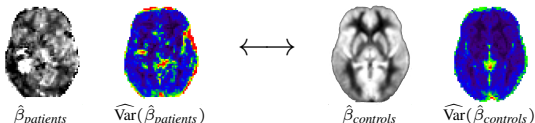
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# Contrast of interest

Camille Maumet



We are interested in the null hypothesis:

$$H_0 : \beta_{controls} = \beta_{patients}. \quad (9)$$

$$H_0 : c \beta_G = 0. \quad (10)$$

Corresponding to the patient versus control group contrast:

$$c = [1 \ -1] \quad (11)$$

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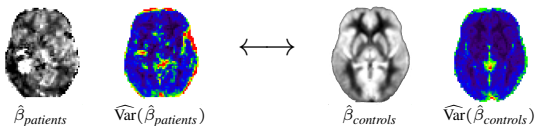
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# Random-effects (RFX) between-group analysis

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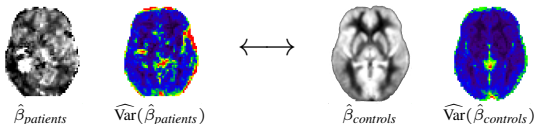
Assuming  $\gamma_{GC,i}^s \sim \mathcal{N}(0, \sigma_{GC,i}^2)$ , by weighted least squares we have:

$$\hat{\beta}_{controls}^{RFX} = \frac{1}{n_1} \sum_{s=1}^{n_1} \hat{\beta}_s, \quad \hat{\beta}_{patients}^{RFX} = \frac{1}{n_2} \sum_{s=n_1+1}^{n_1+n_2} \hat{\beta}_s, \quad (12)$$

The associated sampling variances are:

$$\widehat{\text{Var}}(\hat{\beta}_{controls}^{RFX}) = \frac{\hat{\sigma}_{GC,1}^2}{n_1}, \quad \widehat{\text{Var}}(\hat{\beta}_{patients}^{RFX}) = \frac{\hat{\sigma}_{GC,2}^2}{n_2} \quad (13)$$

# Mixed-effects (MFX) between-group analysis



Assuming  $\gamma_{GC,i}^s \sim \mathcal{N}(0, \sigma_{G,i}^2 + \frac{\sigma_s^2}{r})$ , by weighted least squares we have:

$$\hat{\beta}_{controls}^{MFX} = \frac{1}{\sum_{j=1}^{n_1} w_{j,1}} \sum_{s=1}^{n_1} w_{s,1} \hat{\beta}_s, \quad \text{where } w_{s,i} = \frac{1}{\hat{\sigma}_{G,i}^2 + \frac{\hat{\sigma}_s^2}{r}}$$

$$\hat{\beta}_{patients}^{MFX} = \frac{1}{\sum_{j=n_1+1}^{n_1+n_2} w_{j,2}} \sum_{s=n_1+1}^{n_1+n_2} w_{s,2} \hat{\beta}_s.$$

(14)

The associated sampling variances are:

$$\widehat{Var}(\hat{\beta}_{controls}^{MFX}) = \frac{1}{\sum_{s=1}^{n_1} w_{s,1}}, \quad \widehat{Var}(\hat{\beta}_{patients}^{MFX}) = \frac{1}{\sum_{s=n_1+1}^{n_1+n_2} w_{s,2}}$$

(15)

# Detecting perfusion abnormalities using the GLM

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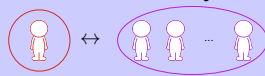
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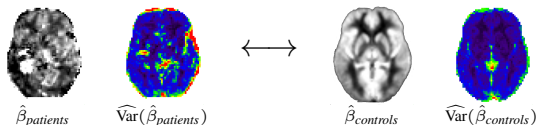
## Between-group analyses



## Individual analyses



# Random-effects (RFX) analysis



Assuming  $\gamma_{GC}^s \sim \mathcal{N}(0, \sigma_{GC}^2)$ , by ordinary least squares we have:

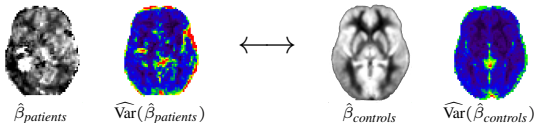
$$\hat{\beta}_{controls}^{RFX} = \frac{1}{n_1} \sum_{s=1}^{n_1} \hat{\beta}_s, \quad \hat{\beta}_{patient}^{RFX} = \hat{\beta}_{n_1+1}, \quad (16)$$

The associated sampling variances are:

$$\widehat{\text{Var}}(\hat{\beta}_{controls}^{RFX}) = \frac{\hat{\sigma}_{GC}^2}{n_1}, \quad \widehat{\text{Var}}(\hat{\beta}_{patient}^{RFX}) = \hat{\sigma}_{GC}^2 \quad (17)$$



## Mixed-effects (MFX) analysis



Assuming  $\gamma_{GC}^s \sim \mathcal{N}(0, \sigma_G^2 + \frac{\sigma_s^2}{r})$ , by weighted least squares we get:

$$\hat{\beta}_{controls}^{MFX} = \frac{1}{\sum_{j=1}^{n_1} w_{j,1}} \sum_{s=1}^{n_1} w_{s,1} \hat{\beta}_s, \quad \hat{\beta}_{patient}^{MFX} = \hat{\beta}_{n_1+1} \quad (18)$$

$$\text{where } w_s = \frac{1}{\hat{\sigma}_G^2 + \frac{\hat{\sigma}_s^2}{r}}.$$

The associated sampling variances are:

$$\widehat{\text{Var}}(\hat{\beta}_{controls}^{MFX}) = \frac{1}{\sum_{s=1}^{n_1} w_s}, \quad \widehat{\text{Var}}(\hat{\beta}_{patients}^{MFX}) = \hat{\sigma}_G^2 + \frac{\hat{\sigma}_{n_1+1}^2}{r} \quad (19)$$

# Random-effects and Mixed-effects analyses

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Control group and patient estimates and sampling variances with RFX and MFX.

	$\hat{\beta}_{controls}$	$\hat{\beta}_{patient}$	$\widehat{Var}(\hat{\beta}_{controls})$	$\widehat{Var}(\hat{\beta}_{patient})$
RFX	$\frac{1}{n_1} \sum_{s=1}^{n_1} \hat{\beta}_s$	$\hat{\beta}_{n_1+1}$	$\frac{\hat{\sigma}_{GC}^2}{n_1}$	$\hat{\sigma}_{GC}^2$
MFX	$\frac{1}{\sum_{j=1}^{n_1} \frac{1}{\hat{\sigma}_G^2 + \frac{\hat{\sigma}_j^2}{r}}} \sum_{s=1}^{n_1} \frac{1}{\hat{\sigma}_G^2 + \frac{\hat{\sigma}_s^2}{r}} \hat{\beta}_s$	$\hat{\beta}_{n_1+1}$	$\frac{1}{\sum_{s=1}^{n_1} \frac{1}{\hat{\sigma}_G^2 + \frac{\hat{\sigma}_s^2}{r}}}$	$\hat{\sigma}_G^2 + \frac{\hat{\sigma}_{n_1+1}^2}{r}$

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# Subjects and imaging protocol

25 patients diagnosed with brain tumours and 61 control subjects participated in this study.

Imaging protocol:

- ▶ PICORE Q2TIPS Pulsed ASL, 60 repetitions
- ▶ MPRAGE T1 3D
- ▶ T2 FLAIR

For the patients only:

- ▶ T1 3D Gadolinium
- ▶ Dynamic Susceptibility Contrast imaging (DSC)

# Validation: Ground Truth

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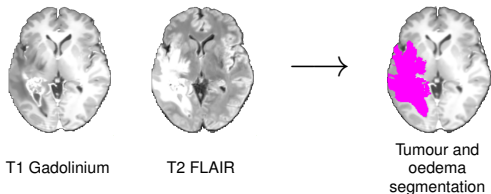
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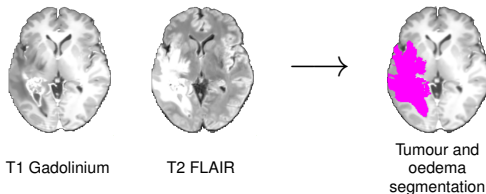
# Validation: Ground Truth

## 1. Segmentation of the tumour:

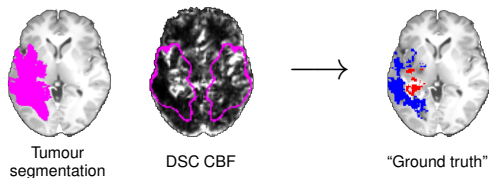


# Validation: Ground Truth

## 1. Segmentation of the tumour:

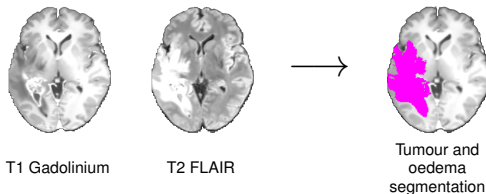


## 2. Combination with T2 perfusion information:

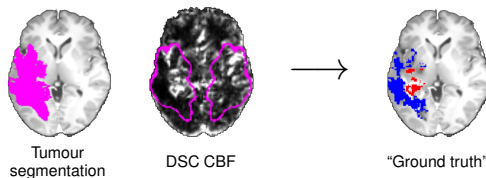


# Validation: Ground Truth

## 1. Segmentation of the tumour:



## 2. Combination with T2 perfusion information:



## 3. Visual assessment and manual corrections by a clinician.



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# Assumptions of RFX analyses

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- ▶ Assumption 1: Within-subject variance negligible by comparison to the between-subject variance.

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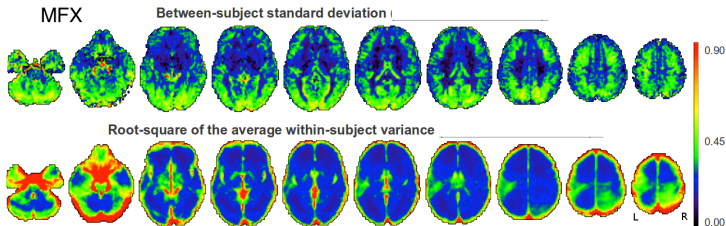
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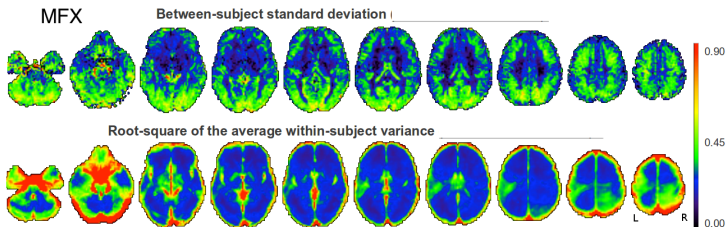
# Assumptions of RFX analyses

- Assumption 1: Within-subject variance negligible by comparison to the between-subject variance.



# Assumptions of RFX analyses

- Assumption 1: Within-subject variance negligible by comparison to the between-subject variance.



× not verified

# Assumptions of RFX analyses

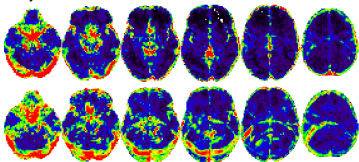
- Assumption 2: Within-subject variance roughly constant across subjects.

# Assumptions of RFX analyses

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- Assumption 2: Within-subject variance roughly constant across subjects.

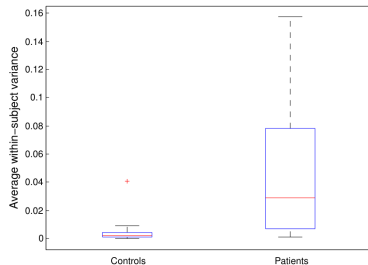
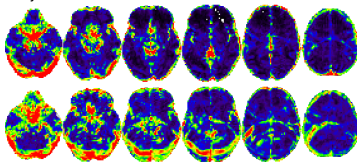
Within-subject standard deviation in two control subjects:



# Assumptions of RFX analyses

- Assumption 2: Within-subject variance roughly constant across subjects.

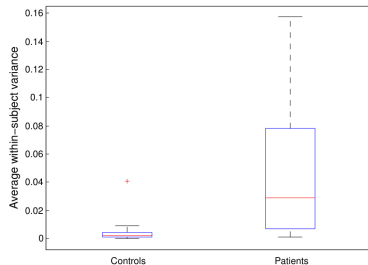
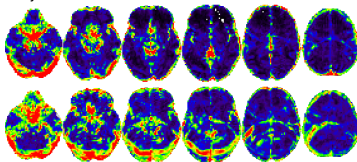
Within-subject standard deviation in two control subjects:



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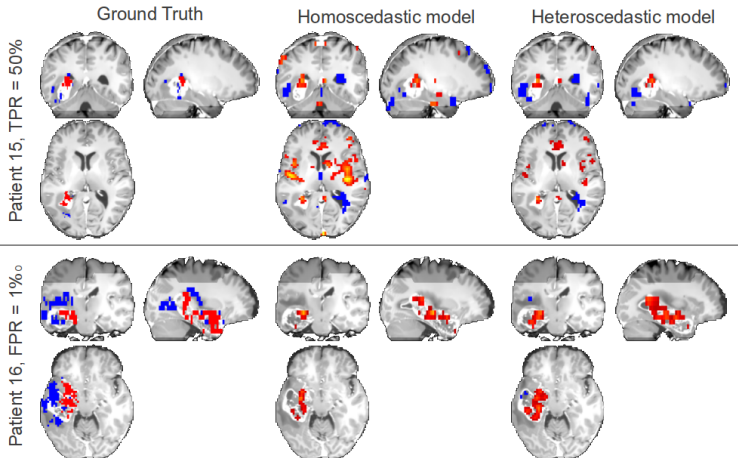
Within-subject standard deviation in two control subjects:



✗ not verified

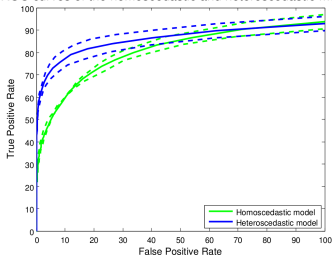


# Results: RFX versus MFX



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ROC curves of the homoscedastic and heteroscedastic models



ROC curves for perfusion abnormality  
detections.

[Maumet et al., NeuroImage 2013]

Random-effects analysis					
FWHM (mm <sup>3</sup> )	0	4	6	8	10
ROC Area	0.46	<b>0.49</b>	<b>0.49</b>	<b>0.49</b>	0.48

Mixed-effects analysis					
FWHM (mm <sup>3</sup> )	0	4	6	8	10
ROC Area	0.63	0.70	<b>0.72</b>	<b>0.72</b>	0.69

Area under the ROC curve.

# Outline

## Detecting perfusion abnormalities

Arterial Spin Labelling

Group analysis

Single-subject analysis

## Random-Effects and Mixed-Effects General Linear Model

Hypothesis testing

Random-effects and Mixed-effects

## Detections in ASL

Methods

Experiment

Results

## Conclusions

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We demonstrated on a dataset of PICORE Q2TIPS ASL images that:

- ▶ the assumptions of RFX analyses were violated.
- ▶ using an **MFX analysis** was essential in the detection of perfusion abnormalities at the patient level.

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- ▶ Further investigation using other ASL sequences are needed.
- ▶ Software packages: **RFX (SPM), MFX (FSL, AFNI)**.
- ▶ More details on RFX and MFX [[Beckmann 2003](#), [Mumford 2006](#), [Mumford 2009](#)].



# Thank you

## VisAGeS team:

- ▶ Christian Barillot.
- ▶ Pierre Maurel.
- ▶ Jean-Christophe Ferré.
- ▶ Béatrice Carsin.

C. Maumet, P. Maurel, J-C. Ferré, B. Carsin, C. Barillot. *Patient-specific detection of perfusion abnormalities combining within-subject and between-subject variances in Arterial Spin Labeling*. NeuroImage, 2013, 81C, pp. 121-130. Freely available online.

Detecting  
perfusion  
abnormalities in  
Arterial Spin  
Labelling

Camille Maumet

Detecting  
perfusion  
abnormalities

ASL

Group

Single-subject

General Linear  
Model: MFX and  
RFX

Hypothesis  
testing

RFX and MFX

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# Q & A

# Outline

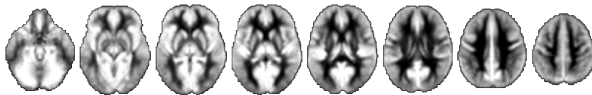
## Appendix

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# A model of normal perfusion

Perfusion estimate  $\hat{\beta}_{controls}$

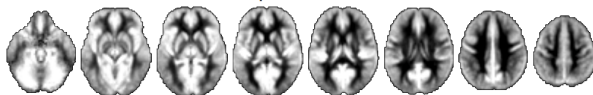
Mean perfusion estimate



# A model of normal perfusion

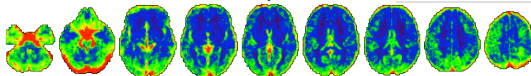
Perfusion estimate  $\hat{\beta}_{controls}$

Mean perfusion estimate

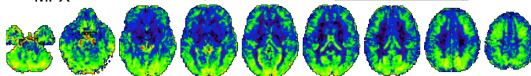


Standard deviation estimates

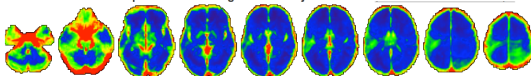
RFX Combined within- and between-subject standard deviation



MFX Between-subject standard deviation



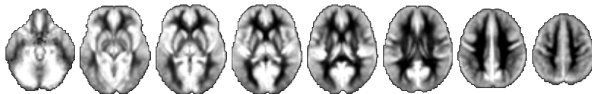
Root-square of the average within-subject variance



# A model of normal perfusion

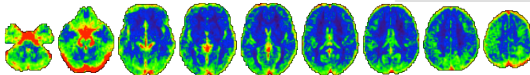
Perfusion estimate  $\hat{\beta}_{controls}$

Mean perfusion estimate

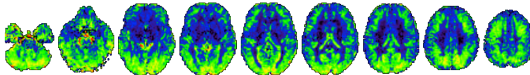


Standard deviation estimates

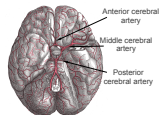
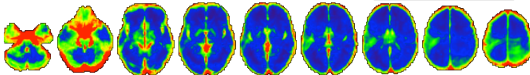
RFX Combined within- and between-subject standard deviation



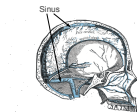
MFX Between-subject standard deviation



Root-square of the average within-subject variance



Brain arteries



Brain veins